$\boxed{3.36}$

$\varphi(x,y) = a\cdot x^2 - a\cdot y^2$

$\varphi(x,y) = a\cdot x\cdot y$

$\vec{E} = -\mathrm{grad}(\varphi)$ $\quad$ $\vec{E} = -\left(\frac{d}{dx}\varphi(x,y)\right)\cdot\vec{i} - \left(\frac{d}{dy}\varphi(x,y)\right)\cdot\vec{j}$

$\overrightarrow{E_1} = -\left[\frac{d}{dx}\left(a\cdot x^2 - a\cdot y^2\right)\right]\cdot\vec{i} - \left[\frac{d}{dy}\left(a\cdot x^2 - a\cdot y^2\right)\right]\cdot\vec{j} \rightarrow E_1 = -2\cdot a\cdot x\cdot i + 2\cdot a\cdot y\cdot j$

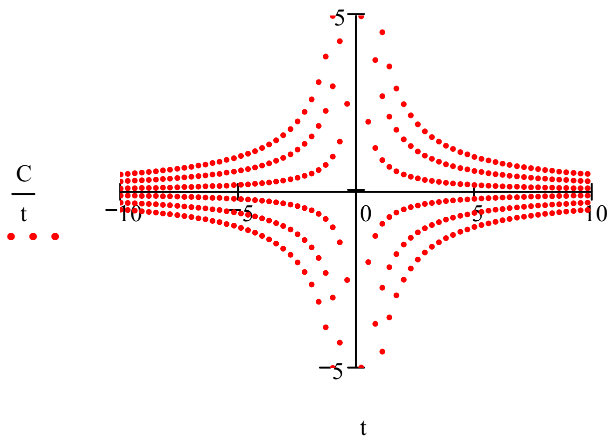
$\overrightarrow{E_1} = -\left(\frac{d}{dx}a\cdot x\cdot y\right)\cdot\vec{i} - \left(\frac{d}{dy}a\cdot x\cdot y\right)\cdot\vec{j} \rightarrow E_1 = -a\cdot y\cdot i - a\cdot x\cdot j$

Линии напряженности - это лини, касательные к которым в данной точке - вектора E, т. е. производная во всех точках совпадает с тангенсом угла наклона E к OX

Пусть: $\overrightarrow{E_1} = I(x,y)\cdot\vec{i} + J(x,y)\cdot\vec{j}$ $\quad$ $y = f(x)$ - линия напряженности $\quad$ $\frac{dy}{dx} = \tan(\varphi) = \frac{J(x,y)}{I(x,y)}$

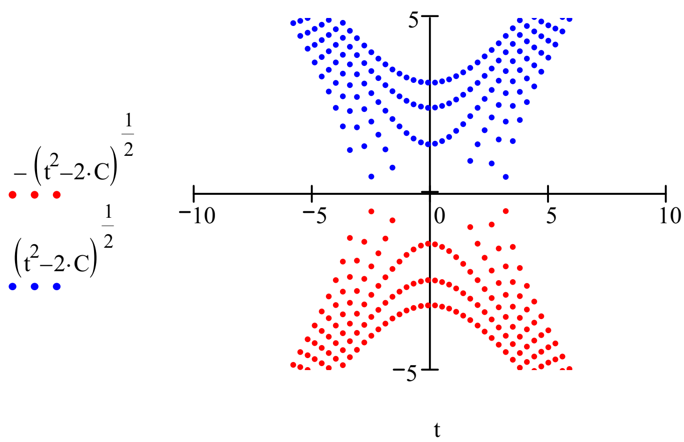
$\frac{dy}{dx} = \frac{-y}{x}$ $\quad$ $\int \frac{1}{y}\,dy = \int \frac{-1}{x}\,dx$ $\quad$ $\ln(y) = \ln\left(\frac{C}{x}\right)$ $\quad$ $y_1 = \frac{C}{x}$

$t := -10, -9.7 .. 10$ $\qquad$ $C := -5, -3 .. 5$



$\frac{dy}{dx} = \frac{x}{y}$ $\quad$ $\int y\,dy = \int x\,dx$ $\quad$ $\frac{x^2}{2} - \frac{y^2}{2} = C$ $\quad$ $y = \left(x^2 - 2\cdot C\right)^{\frac{1}{2}}$ $\quad$ $y = -\left(x^2 - 2\cdot C\right)^{\frac{1}{2}}$

$t := -10, -9.7 .. 10$



3.48

$$\vec{E} = I(x,y,z)\cdot\vec{i} + J(x,y,z)\cdot\vec{j} + K(x,y,z)\cdot\vec{k}$$

$$\vec{E_1} = a\cdot\left(y\cdot\vec{i} + x\cdot\vec{j}\right)$$

$$\vec{E_2} = 2\cdot a\cdot x\cdot y\cdot\vec{i} + a\cdot\left(x^2 - y^2\right)\cdot\vec{j}$$

$$\vec{E_3} = a\cdot y\cdot\vec{i} + (a\cdot x - b\cdot z)\cdot\vec{j} + b\cdot y\cdot\vec{k}$$

$$\varphi = -\int_{(0,0,0)}^{(x,y,z)} I(x,y,z)\cdot dx + J(x,y,z)\cdot dy + K(x,y,z)\,dz$$

$$\varphi = -\int_{(0,0,0)}^{(x,0,0)} I(x,y,z)\cdot dx + J(x,y,z)\cdot dy + K(x,y,z)\,dz - \int_{(x,0,0)}^{(x,y,0)} I(x,y,z)\cdot dx + J(x,y,z)\cdot dy + K(x,y,z)\,dz - \int_{(x,y,0)}^{(x,y,z)} I(x,y,z)\cdot dx + J(x$$

$y = 0$ $\quad$ $dx = 0$ $\quad$ $dx = 0$

$z = 0$ $\quad$ $z = 0$ $\quad$ $dy = 0$

$$\varphi = -\int_0^x I(x,y,z)\,dx - \int_0^y J(x,y,z)\,dy - \int_0^z K(x,y,z)\,dz$$

$y = 0$ $\quad$ $x = x$ $\quad$ $x = x$

$z = 0$ $\quad$ $z = 0$ $\quad$ $y = y$

$$\vec{E_1} = a\cdot\left(y\cdot\vec{i} + x\cdot\vec{j}\right)$$

$$\varphi_1 = -\int_0^x a\cdot y\,dx - \int_0^y a\cdot x\,dy = -a\cdot x\cdot y + C$$

$y = 0$ $\quad$ $x = x$

$$\vec{E_2} = 2\cdot a\cdot x\cdot y\cdot\vec{i} + a\cdot\left(x^2 - y^2\right)\cdot\vec{j}$$

$$\varphi_2 = -\int_0^x 2\cdot a\cdot x\cdot y\,dx - \int_0^y a\cdot\left(x^2 - y^2\right)dy = -\left(a\cdot x^2\cdot y - \frac{1}{3}\cdot a\cdot y^3\right) + C$$

$y = 0$ $\quad$ $x = x$

$$\vec{E_3} = a\cdot y\cdot\vec{i} + (a\cdot x - b\cdot z)\cdot\vec{j} + b\cdot y\cdot\vec{k}$$

$$\varphi_3 = -\int_0^x a\cdot y\,dx - \int_0^y (a\cdot x - b\cdot z)\,dy - \int_0^z b\cdot y\,dz = -\int_0^y a\cdot x\,dy - \int_0^z b\cdot y\,dz = -y\cdot a\cdot x - z\cdot b\cdot y$$

$y = 0$ $\quad$ $x = x$ $\quad$ $x = x$

$z = 0$ $\quad$ $z = 0$ $\quad$ $y = y$

3.49

$$\varphi(x) = -a\cdot x^3 + b \qquad \frac{d^2}{dx^2}\varphi + \frac{d^2}{dy^2}\varphi + \frac{d^2}{dz^2}\varphi = \frac{-\rho}{\varepsilon_0} \qquad \rho = -\varepsilon_0\cdot\frac{d^2}{dx^2}\varphi(x) = -\varepsilon_0\cdot\frac{d^2}{dx^2}\left(-a\cdot x^3 + b\right) = 6\cdot\varepsilon_0\cdot a\cdot x$$

$\rho(x)$

3.51

$$\varphi(r) = a\cdot r^2 + b \qquad \frac{d^2}{dx^2}\varphi + \frac{d^2}{dy^2}\varphi + \frac{d^2}{dz^2}\varphi = \frac{-\rho}{\varepsilon_0}$$

$\rho(r)$

$$r = \sqrt{x^2 + y^2 + z^2} \qquad \varphi(x,y,z) = a\cdot\left(\sqrt{x^2 + y^2 + z^2}\right)^2 + b = a\cdot\left(x^2 + y^2 + z^2\right) + b$$

$$\frac{d^2}{dx^2}\left[a\cdot\left(x^2 + y^2 + z^2\right) + b\right] + \frac{d^2}{dy^2}\left[a\cdot\left(x^2 + y^2 + z^2\right) + b\right] + \frac{d^2}{dz^2}\left[a\cdot\left(x^2 + y^2 + z^2\right) + b\right] \rightarrow 6\cdot a \qquad \frac{-\rho}{\varepsilon_0} = 6\cdot a \qquad \rho = -6a\cdot\varepsilon_0$$